

# Data-driven optimization of the aeroelastic performance of long-span suspension bridge based on Bayesian modeling

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## SUMMARY:

Designing long-span bridges to withstand wind loads has become the principal concern of designers and scholars. Two methods are used to assess the aerodynamic performance of long-span bridges: (i) Wind tunnel tests and (ii) Computational fluid dynamics (CFD) simulations. While wind tunnel tests have been the only method to assess the aerodynamic performance of long-span bridges, the recent trend in bridge aerodynamic study is increasingly relying on CFD simulations. However, both CFD simulation and wind tunnel tests are time-consuming. To tackle this flaw, scholars have proposed machine learning (ML) models. Despite the prowess of ML models in the wind engineering field, they provide accurate results only when big data are available which hinders their application since generating big data in wind engineering is cost prohibitive. Additionally, both wind tunnel test and CFD data are prone to uncertainties which are necessary to quantify during the prediction. This study proposes a hierarchical Bayesian modeling (HBM) to predict the critical flutter velocity of the long-span bridge, which overcomes the weaknesses of the conventional ML approaches.

**Keywords:** *Long-span bridge, Flutter velocity, Hierarchical Bayesian modeling, Uncertainties quantification, accuracy*

## 1. INTRODUCTION

Understanding the aeroelastic behavior of long-span bridges is crucial for the wind-resistant design and analysis of such mega structures (Tinmitondé et al., 2022). The growing main span of these structures makes them very sensitive to wind-induced vibration, especially flutter instability. Therefore, robust and accurate estimation models are needed to overcome the potential destruction of long-span bridges due to the flutter phenomenon (Scanlan, 1990). Wind tunnel tests and CFD simulations are commonly used to study the aeroelastic behavior of long-span bridges. However, the aforementioned approaches are time-consuming and cost-prohibitive. Therefore, scholars are now turning to data-driven models such as surrogate models and machine learning models (Kareem, 2020; Rizzo and Caracoglia, 2020). However, the machine learning algorithms used to predict the flutter velocity were built using deterministic structural and aerodynamic parameters without considering the uncertainties in variables (geometric properties, and aerodynamic derivatives). Besides, machine learning models need significant data for accurate prediction, which limit their application in the wind engineering field.

This paper aims at proposing hierarchical Bayesian modeling to improve the prediction accuracy

of the critical flutter velocity by including the uncertainties in the data.

## 2. THEORETICAL BACKGROUND

### 2.1 Hierarchical Bayesian model

The proposed Bayesian inference consists of three steps (See Fig. 1). Firstly, the distribution of the prior uncertain parameters is given. Secondly, based on Bayes's theorem, the posterior distribution is calculated. Finally, the unobserved data point is determined by using the posterior distribution.

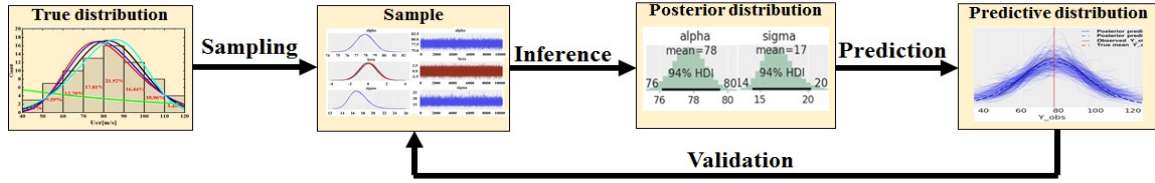


Fig. 1 Roadmap of the hierarchical Bayesian modeling framework

### 2.2 Mathematical construct of hierarchical Bayesian model

The relationship between the response and independent variables can be expressed as follows (see Eq. (1)):

$$y = \alpha + \beta X + \varepsilon \quad (1)$$

where  $y$  is the target,  $X$  is the variables,  $\alpha$  and  $\beta = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)$  are unknown coefficients parameters to be estimated.  $\varepsilon$  is the error.

From a probability distribution viewpoint, Eq. (1) can be rewritten as follow (see Eq. (2)):

$$f(y|\alpha, \beta, \sigma_\varepsilon) = \rho\left(\frac{y - \alpha - \beta X}{\sigma_\varepsilon}\right) \quad (2)$$

where  $\rho$  is the probability density function (PDF) of the normal distribution.

Assuming that all data points are statistically independent identically distribution (i.i.d). Then the probability distribution can be expressed as follow (see Eq. (3)):

$$f(D|\alpha, \beta, \sigma_\varepsilon) = \prod_i^n f(x_i, y_i|\alpha, \beta, \sigma_\varepsilon) \quad (3)$$

By denoting  $\theta$  the random variables to be estimated in the flutter velocity of long-span suspension bridges.  $\pi(\mu_\alpha), \pi(\mu_\beta), \pi(\sigma_\alpha), \pi(\sigma_\beta), \pi(\sigma_\varepsilon)$  are the prior probability density distribution of the hyperparameters  $\mu_\alpha, \mu_\beta, \sigma_\alpha, \sigma_\beta, \sigma_\varepsilon$ . Then, Eq. (4) can be obtained.

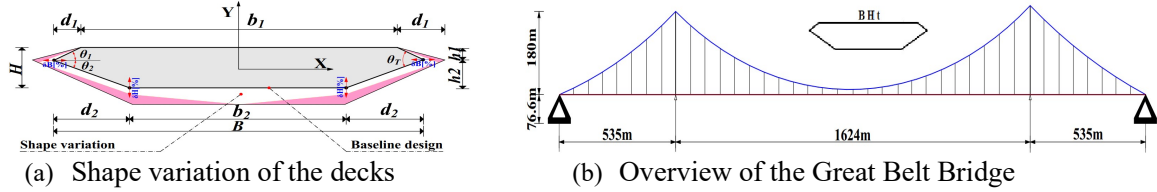
$$f(\theta|D) \propto \frac{1}{2\pi\sqrt{2\pi\sigma_\alpha^2\sigma_\beta^2}} \pi(\mu_\alpha)\pi(\mu_\beta)\pi(\sigma_\alpha)\pi(\sigma_\beta)\pi(\sigma_\varepsilon) \exp\left[-\frac{1}{2}\left[\left(\frac{\alpha - \mu_\alpha}{\sigma_\alpha}\right)^2 + \left(\frac{\beta - \mu_\beta}{\sigma_\beta}\right)^2\right]\right] \exp\left[-\frac{1}{2}\sum_{i=1}^n \left[\left(\frac{y_i - \alpha - \beta^T X_i}{\sigma_\varepsilon}\right)^2\right]\right] \quad (4)$$

## 3. APPLICATION TO LONG-SPAN BRIDGE FLUTTER VELOCITY PREDICTION

### 3.1 Data collection strategy

A series of 73 designs of experiments (DoE) was generated from a streamlined deck section using a uniform sampling method as discussed in our previous work (Tinmitondé et al., 2022). The force coefficients and their derivatives were calculated for each sample design using 2D-

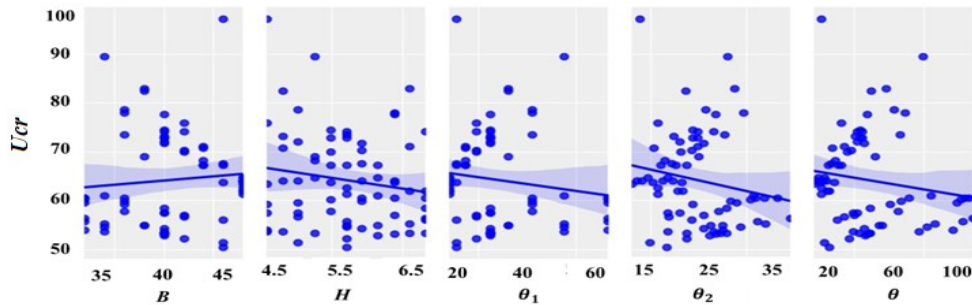
URANS CFD simulations. The critical flutter velocity of each DoE was then computed using a quasi-steady theory by using the dynamic parameters of a Great Belt Bridge. The shape configuration of the deck cross-section as well as the overview of the great can be seen in **Fig. 2**.



**Fig. 2** Shape variation of the streamlined deck under study and the overview of the Great Belt Bridge

### 3.2 Analyse of the data used to train the Bayesian model

To train the Bayesian model the most influential design parameters  $B, H, \theta_1, \theta_2, \theta_T$  of the streamlined bridge deck were chosen as input variables and the critical flutter velocity was considered the output variable. **Fig. 3** shows the relationship between the design parameters and the critical flutter velocity. Furthermore, to better understand which probability distribution describes our critical flutter velocity data, several probabilities (normal, lognormal, exponential, and Weibull) distribution using two statistical tests such as Anderson Darling test and the Kolmogorov-Smirnov test. The results indicated that both normal and lognormal distributions satisfied the statistical tests and therefore described better the flutter velocity data. Nonetheless, only normal distribution is retained to conduct this study.

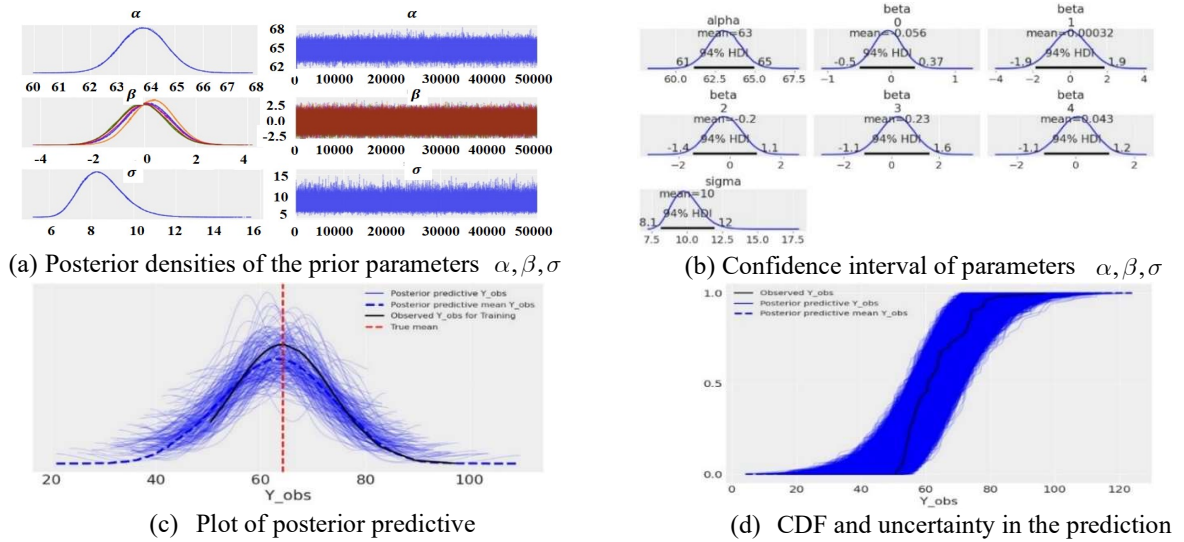


**Fig. 3** Relationship between inputs and output variables

## 4. RESULTS AND DISCUSSIONS

To perform the proposed Bayesian model in this study, 50000 samples of  $\mu_\alpha, \mu_\beta, \sigma_\alpha, \sigma_\beta, \sigma_\epsilon, \alpha, \beta$  were drawn for the training and testing set (70% and 30% respectively) using MCMC (Markov Chain Monte Carlo) simulation methods to predict the posterior probability distribution of  $\theta$  which was implemented in *Python 3.8*. Three MCMC sampling techniques (NUTS, Metropolis, and Hamiltonian MC) were used and the results indicated that the sampling technique adopted can affect the accuracy of the prediction results. Moreover, four chains were used to ensure that the accuracy of the results was not affected by the number of chains during the simulations. The results of 50000 samples of  $\alpha, \beta$ , and  $\sigma_\epsilon$  were generated based on the MCMC simulation for both training and testing data, and only the result of the training is presented here for brevity. The results indicated that the model with NUTS sampling

presented the best performance on both the training and testing data. **Fig. 4** shows the training results of the Bayesian model using the NUTS sampling strategy. Furthermore, the proposed Bayesian model was compared to three conventional ML models (support vector regression, random forest, and extreme gradient boosting), and the results of the performance metrics show that the Bayesian model overperformed the three ML models. Furthermore, more details can be found on the same research previously conducted by the authors where sensitivity and correlation analyses between design parameters were discussed (Tinmitondé et al., 2023).



**Fig. 4** Training set results for  $N=50000$  using NUTS sampler, and the running time is  $T=759$ seconds

## 5. CONCLUSIONS

Bayesian modeling was used to predict the critical flutter velocity of a long-span suspension bridge where the five influential design variables were used as inputs. The results indicated that the proposed Bayesian model overperformed the conventional machine learning models and can include uncertainties in the data during prediction. Nonetheless, the phenomenon of blessing and curse of uncertainties is already included in our future research for further validation.

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